محاضرة رياضة

- مسألة آخر محاضرة :-

$6x - x^2 \longrightarrow$ إكمال مربع

$$I = \int_3^{\infty} e^{6x - x^2}\, dx$$

(جذر الأول الإشارة $\frac{1}{2}$ معامل الثاني)^2 - ($\frac{1}{2}$ معامل الثاني)^2

$$6x - x^2 = -[x^2 - 6x] = -[(x-3)^2 - 9]$$

$$I = e^{9} \int_3^{\infty} e^{-(x-3)^2}\, dx \qquad (x-3)^2 = t$$

$$x - 3 = t^{\frac{1}{2}} \Rightarrow dx = \frac{1}{2} t^{-\frac{1}{2}}\, dt$$

$$I = \frac{1}{2} e^{9} \int_0^{\infty} t^{-\frac{1}{2}} e^{-t}\, dt$$

$$= \frac{1}{2} e^{9}\, \Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}\, e^{9}}{2}$$

Beta Function

$\boxed{1}$ $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

$\boxed{2}$ $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$

$\boxed{3}$ $\beta(m,n) = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$

$\boxed{4}$ $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$

$\boxed{5}$ $\beta(m,n) = \beta(n,m)$

← هذا الجزء يجعلنا نحسب التكاملات على الصور من (١ ← ٣)

بمجرد النظر عليها

$$\int_0^1 x^2(1-x)^3\,dx = \beta(3,4) = \frac{\Gamma(3)\,\Gamma(4)}{\Gamma(7)} = \frac{(2!)(3!)}{6!}$$

Lec22 $\boxed{2}$

$\boxed{\text{الأقرى كار}}$

الأفكار

[1] حدود التكامل ليست من (0 ← 1) وداخل القوس ليس $(1-x)$ } ← نجهز ما بداخل القوس ونجعله (رمز - 1) والرمز ليس له أس وليس له معاملات ونستخدم الصورة رقم (1) .

[2] دوال مثلثية ← نستخدم الصورة رقم (2)

a) إذا كانت الحدود $\left(\int_0^{\pi}\right) \Leftarrow \int_0^{\pi} = 2\int_0^{\frac{\pi}{2}}$

إذا كان $f(x+\frac{\pi}{2}) = -f(x)$

b) إذا تغيرت الزاوية نستخدم حساب المثلثات .

[3] إذا كانت الحدود تحتوي على ∞ نجهز ما بداخل القوس ونستخدم رقم (3) .

Ex Evaluate

① $\int_0^1 x^4 (1-x)^8 \, dx$

② $\int_0^1 (x^3-\sqrt{x})^2 (\sqrt{1-x}) \, dx$

③ $\int_0^1 \sqrt{\frac{1}{x}-1} \, dx$

④ $\int_0^4 x^2 \sqrt{4-x} \, dx$

⑤ $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$

⑥ $\int_2^5 (x-2)\sqrt{5-x} \, dx$

⑦ $\int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^3\theta}{\cos\theta}}$ ⑧ $\int_0^{\infty} \frac{dx}{1+x^4}$

$\boxed{9}$ $\int_0^{\frac{\pi}{2}} \sin^p\theta \, d\theta = \begin{cases} \frac{2n!\,\pi}{2^n (n!)^2 * 2} ; & P = 2n \\ \frac{n\, 2^{2n-1} (n!)^2}{2n!} & P = 2n-1 \end{cases}$

$P > 0$

Sol

$\boxed{1}$ $m-1=4 \,,\, n-1=8 \Rightarrow m=5 \,,\, n=9$

$I = \beta(5,9) = \frac{\Gamma(5)\,\Gamma(9)}{\Gamma 14} = \frac{4!\; 8!}{13!}$

$\boxed{2}$

$I = \int_0^1 (x^6 - 2x^{\frac{7}{2}} + x)(1-x)^{\frac{1}{2}} dx$

$= \underbrace{\int_0^1 x^6 (1-x)^{\frac{1}{2}} dx}_{m=7 \,,\, n=\frac{3}{2}} - \underbrace{\int_0^1 2x^{\frac{7}{2}} (1-x)^{\frac{1}{2}} dx}_{m=\frac{9}{2} \,,\, n=\frac{3}{2}} + \underbrace{\int_0^1 x (1-x)^{\frac{1}{2}} dx}_{m=2 \quad n=\frac{3}{2}}$

$= \frac{\Gamma(7)\,\Gamma(\frac{3}{2})}{\Gamma(\frac{17}{2})} - 2\frac{\Gamma(\frac{9}{2})\,\Gamma(\frac{3}{2})}{\Gamma(6)} + \frac{\Gamma(2)\,\Gamma(\frac{3}{2})}{\Gamma(\frac{7}{2})}$

$$\Gamma\left(\tfrac{3}{2}\right) = \tfrac{1}{2}\Gamma\left(\tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\Gamma\left(\tfrac{9}{2}\right) = \left(\tfrac{7}{2}\right)\left(\tfrac{5}{2}\right)\left(\tfrac{3}{2}\right)\frac{\sqrt{\pi}}{2} = \frac{105\sqrt{\pi}}{16}$$

$$\Gamma\left(\tfrac{7}{2}\right) = \left(\tfrac{5}{2}\right)\left(\tfrac{3}{2}\right)\frac{\sqrt{\pi}}{2} = \frac{15\sqrt{\pi}}{8}$$

$$\Gamma\left(\tfrac{17}{2}\right) = \left(\tfrac{15}{2}\right)\left(\tfrac{13}{2}\right)\left(\tfrac{11}{2}\right)\left(\tfrac{9}{2}\right)\frac{105\sqrt{\pi}}{16}$$

$\Gamma(7) = 6!$ ، $\Gamma(6) = 5!$ ، $\Gamma(2) = 1!$

بالتعويض ع *

$\boxed{3}$ $$\int_0^1 \sqrt{\frac{1}{x} - 1}\, dx = \int_0^1 \sqrt{\frac{1-x}{x}}\, dx$$

$$= \int_0^1 \sqrt{\frac{1}{x}(1-x)}\, dx = \int_0^1 \left(\frac{1}{x}\right)^{\frac{1}{2}} (1-x)^{\frac{1}{2}}\, dx$$

$\vdots$ أكمل $\vdots$

$\boxed{5}$ Lec 22

④ $\int_0^4 x^4 \sqrt{4-x}\, dx$

$$I = 2 \int_0^4 x^2 \left(1 - \frac{x}{4}\right)^{\frac{1}{2}} dx$$

$t = \frac{x}{4} \Rightarrow x = 4t \quad , \quad dx = 4\,dt$

at $x = 0 \longrightarrow x = \frac{0}{4} = 0$

at $x = 4 \longrightarrow x = \frac{4}{4} = 1$

$$I = 2 \int_0^1 16\, t^2 (1-t)^{\frac{1}{2}}\, 4\, dt$$

$$= 128 \int_0^1 t^2 (1-t)^{\frac{1}{2}}\, dt = 128\, \beta\left(3, \frac{3}{2}\right)$$

$$= 128 \frac{\Gamma(3)\, \Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{9}{2}\right)} = \frac{(128)(2!)\, \Gamma\left(\frac{3}{2}\right)}{\left(\frac{7}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)}$$

$\boxed{5}$ Put $t = x^n$ هتلاقي الورقة

$\boxed{6}$ Lec22

$\boxed{6}$ $I = \int_{2}^{5} (x-2)\sqrt{5-x}\, dx$

$t = x-2 \longrightarrow x = t+2 \longrightarrow dx = dt$

$x = 2 \longrightarrow t = 0$

$x = 5 \longrightarrow t = 3$

$I = \int_{0}^{3} t\sqrt{5-t-2}\, dt = \int_{0}^{3} t\sqrt{3-t}\, dt$

$= \sqrt{3}\int_{0}^{3} t\left(1-\frac{t}{3}\right)^{\frac{1}{2}} dt$

$y = \frac{t}{3} \Longrightarrow$ $t = 0 \longrightarrow y = 0$

$t = 3 \longrightarrow y = 1$

$I = 9\sqrt{3}\int_{0}^{1} y(1-y)^{\frac{1}{2}}\, dy$

$= 9\sqrt{3}\,\beta\left(2, \frac{3}{2}\right) = 9\sqrt{3}\,\frac{\Gamma(2)\,\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{7}{2}\right)}$

$\boxed{7}$ Lec 22

$\boxed{7}$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^3\theta}{\cos\theta}} = \int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}}\theta \, \cos^{\frac{-1}{2}}\theta$$

$$2m-1 = \frac{3}{2}$$

$$2n-1 = \frac{-1}{2}$$

أكمل

$\boxed{8}$ $\int_0^{\infty} \frac{dx}{(1+x^4)} \implies \text{Put } x^4 = y$

$\boxed{9}$

$$I = \int_0^{\frac{\pi}{2}} \sin^{p}\theta \; \cancel{\text{and}} * (\cos\theta)^0 \, d\theta$$

$$\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$$

$$2m-1 = p \quad , \; 2n-1 = 0$$

$$m = \frac{p+1}{2} \quad , \; n = \frac{1}{2}$$

$$I = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{1}{2}\right).$$

$\boxed{8}$ Lec 22

$$I = \frac{1}{2} \frac{\Gamma\left(\frac{P+1}{2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{P+2}{2}\right)}$$

$P = 2n$

$$I = \frac{\Gamma\left(n+\frac{1}{2}\right)\sqrt{\pi}}{n!} \longrightarrow (1)$$

$$\Gamma\left(n+\frac{1}{2}\right) = \left(n-\frac{1}{2}\right)\Gamma\left(n-\frac{1}{2}\right)$$

$$= \left(n-\frac{1}{2}\right)\left(n-\frac{3}{2}\right)\left(n-\frac{5}{2}\right) \cdots\cdots \Gamma\left(\frac{1}{2}\right)$$

$$= \left(\frac{2n-1}{2}\right)\left(\frac{2n-3}{2}\right)\left(\frac{2n-5}{2}\right) \cdots \sqrt{\pi}$$

ح لكي نظهر $2n!$ نضرب في الحدود الناقصة بسطاً ومقاماً.

$$= \frac{2n}{2n}\left(\frac{2n-1}{2}\right)\left(\frac{2n-2}{2n-2}\right)\left(\frac{2n-3}{2}\right)\left(\frac{2n-4}{2n-4}\right) \cdots \sqrt{\pi}$$

$$= \frac{2n!\sqrt{\pi}}{2^n * 2^n * n(n-1)(n-2)\cdots} = \frac{2n!\sqrt{\pi}}{2^{2n}\, n!}$$

بالتعويض في (1)

بالتعويض في (1)

$$I = \frac{(2n!)\,\pi}{2\,2^{2n}\,(n!)^2} \longrightarrow \# \ \boxed{a}$$

$P = 2n - 1$

$$I_2 P = \frac{1}{2}\,\frac{\Gamma\left(\frac{P+1}{2}\right)\sqrt{\pi}}{\Gamma\left(\frac{P+2}{2}\right)} = \frac{\Gamma(n)\,\sqrt{\pi}}{\Gamma\left(n+\frac{1}{2}\right)}$$

تم حسابها في الجزئية السابقة

$$I_2 = \frac{(n-1)!\,\sqrt{\pi}}{2\,(2n!)\,\sqrt{\pi}} * 2^{2n} * n!$$

باقي في هذا الجزء أن نتعرف على أسلوب إستنتاج الصور 2، 3، 4 من (1).

$\boxed{Ex:1}$ show that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$

$\boxed{Sol}$

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Let $x = \sin^2\theta$, $dx = 2\sin\theta\cos\theta\,d\theta$

at $X=0 \longrightarrow \theta=0$

at $X=1 \longrightarrow \sin\theta=1 \Rightarrow \theta=\frac{\pi}{2}$

$$I=\int_0^{\frac{\pi}{2}}(\sin^2\theta)^{m-1}(1-\sin^2\theta)^{n-1} * 2\sin\theta\cos\theta \, d\theta$$

$$I=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

Ex:2 show that $\beta(m,n)=\int_0^{\infty}\frac{y^{m-1}}{(1+y)^{m+n}}dx$

$$\beta(m,n)=\int_0^1 X^{m-1}(1-x)^{n-1}dx$$

أسلوب التفكير ← نريد جعل الحدود القديمة كالجديدة حتى

$1 \rightarrow \infty$ لابد أخذ تعويضة:

$$X=\frac{y}{1+y}$$

$\frac{\text{رقم}}{1+\text{رقم}}$ يعطى (1) وهذا لايمكن حدوثه إلا إذا كانت الرقم (∞)

at $X=0 \Rightarrow 0=\frac{y}{1+y} \Rightarrow y=0$

at $X=1 \Rightarrow 1=\frac{y}{1+y} \Rightarrow \lim_{y\to\infty}\frac{y}{1+y}=1$

$\Rightarrow y \longrightarrow \infty$

$$dx = \frac{(1+y)(1) - y(1)}{(1+y)^2}\, dy = \frac{dy}{(1+y)^2}$$

$$I = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m-1}} \left(1 - \frac{y}{(1+y)}\right)^{n-1} * \frac{dy}{(1+y)^2}$$

$$\frac{1+y-y}{1+y} \rightarrow \left(\frac{1}{1+y}\right)^{n-1}$$

$$I = \int_0^{\infty} \frac{y^{m-1}\, dy}{(1+y)^{m-1+n-1+2}}$$

$$I = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}}\, dy$$